Fluid Friction in Noncircular Ducts

J. E. WALKER, G. A. WHAN, and R. R. ROTHFUS

Carnegie Institute of Technology, Pittsburgh, Pennsylvania

Pressure drop due to fluid friction has been measured in a smooth tube; in six smooth concentric annuli; and between smooth, parallel, flat plates. The data cover the viscous, transition, and lower turbulent ranges of flow. Friction factors are in agreement with theory in the viscous range and can be correlated uniquely in the fully turbulent range by means of a modified hydraulic radius concept. Limits of the transition region in annuli are functions of the ratio of inner and outer radii. Correlations in the fully turbulent range permit friction factors for the noncircular ducts to be predicted within the uncertainty of smooth-tube data. Such factors can be used for the purpose of correlating other variables as well as for direct calculation of pressure drop.

The characteristics of fluid friction in smooth tubes are well known through many experimental studies. Particularly in the realm of isothermal flow, the empirical data are sufficient to establish the limits of the viscous, transition, and turbulent regimes as well as the Reynolds-number effect on friction within each region.

Unfortunately, the same is not true for isothermal flow in noncircular conduits. Taken in total, the data are numerous, but none of the common cross sections, considered individually, has been investigated as thoroughly as the circular tube. It is the purpose of this paper to present friction data for flow in concentric annuli and between parallel plates which are as consistent and precise as typical tube data and numerous enough to be meaningful.

The present investigation deals with the isothermal, steady, uniform flow of water at room temperature. Pressure-drop measurements have been made in conduits having only uniformly smooth brass, copper, and steel surfaces. The data cover the viscous, transition, and lower turbulent ranges of flow in a tube, in six concentric annuli of various radius ratios, and between parallel flat plates.

PRELIMINARY DISCUSSION

Basic Relationships

The pressure drop due to friction in a tube or pipe can be represented by means of the Fanning equation

$$\Delta p = \frac{2f\rho V^2 L}{g_0 D} \tag{1}$$

Dimensional considerations indicate that the friction factor f should be a function of the Reynolds number alone in tubes of negligible roughness. It is usual to define the Reynolds number in the manner

$$N_{Re} = \frac{DV\rho}{\mu} \tag{2}$$

largely as a matter of convenience. Other characteristic lengths and velocities could be chosen in establishing a criterion for the onset of turbulent flow if it should prove advantageous to do so.

In dealing with steady, uniform flow through uniform conduits of noncircular cross section, it is customary to write Equations (1) and (2) with an "equivalent diameter" D_e replacing the tube diameter D. By analogy with the tube, the equivalent diameter is taken to be four times the cross-sectional area occupied by the fluid divided by the wetted perimeter on which the fluid exerts skin friction. Since the latter ratio is known as the *hydraulic radius* of the conduit,

$$D_e = 4R_H \tag{3}$$

Therefore, for tubes and noncircular conduits alike,

$$N_{Re} = \frac{D_e V\rho}{\mu} \tag{4}$$

and

$$\Delta p = \frac{2f\rho V^2 L}{g_0 D_e} \tag{5}$$

The problem, of course, is to set forth the correct relationship between the friction factor and Reynolds number within each stable flow regime for each type of conduit. In the absence of an adequate theory of turbulence, the experimental problem is enormous. It, therefore, becomes a practical nesessity to seek correlations of fluid friction in noncircular ducts with that in tubes at predetermined conditions. In the realm of fully viscous flow, theoretical expressions for the friction factors can be developed on the basis of zero slip at the conduit walls.

Geometrical Considerations

Smooth, concentric annuli have certain characteristics which prove advantageous in the study of fluid friction. Symmetry dictates that the skin friction should be uniform over either of the two boundaries taken separately. On the other hand, the skin friction on the inner surface, or core, is greater than that on the outer surface by an amount depending on the ratio of the surface radii. Thus geometrical differences can be studied without the additional complication of point-to-point variations in the boundary conditions. Furthermore, from the standpoint of friction, tubes and parallel plates are limiting cases of annular cross sections; therefore, all the conduits studied in the present investigation can be considered annuli of various radius ratios.

When the entire stream is taken as the basis of definition, the equivalent diameter of an annulus is simply the difference in diameters of the outer and inner boundaries, that is,

$$D_e = D_2 - D_1 \tag{6}$$

G. A. Whan is at present at the University of New Mexico, Albuquerque, New Mexico.

For the limiting case of tubes, where the core is made vanishingly small, $D_e = D_2$. On the other hand, for the limiting case of parallel plates of infinite extent $D_e = 4b$ where b is the half clearance between the plates.

Viscous Flow

When the flow in a concentric annulus is entirely viscous, theory predicts that combination of Equations (4), (5), and (6) should yield

$$f = \frac{16}{N_{Re}} \psi_1\left(\frac{r_1}{r_2}\right) \tag{7}$$

$$= \frac{16}{N_{Re}} \left[\frac{1 - 2\left(\frac{r_1}{r_2}\right) + \left(\frac{r_1}{r_2}\right)^2}{1 + \left(\frac{r_1}{r_2}\right)^2 + \dfrac{1 - \left(\frac{r_1}{r_2}\right)^2}{\ln\left(\frac{r_1}{r_2}\right)}} \right]$$

For a tube the bracketed term is 1.00 and for parallel plates it is 1.50. Values at intermediate radius ratios can be obtained from Table 1.

Senecal and Rothfus (*10*) measured pressure gradients in smooth tubes and found Equation (7) to be obeyed exactly at Reynolds numbers of less than 1,200. Prengle and Rothfus (*7*), using a dye technique, found viscous flow at every point in smooth tubes when the Reynolds number was less than 1,225. On the other hand, their results for concentric annuli indicate an initial deviation from entirely viscous flow at Reynolds numbers in the vicinity of 700. The pressure-drop data of Carpenter and coworkers (*2*) for one annulus confirm Equation (7) up to a Reynolds number of about 800.

Transition Flow

Senecal and Rothfus have demonstrated that the reproducibility of transition-range friction factors in tubes can be made equal to that in the fully turbulent range with sufficient care. The only other requirement is a normally high level of disturbance in the tube entry.

At Reynolds numbers between 1,200 and 2,030 in smooth tubes Senecal and Rothfus observed a slight progressive departure from the friction-factor equation for viscous flow. The principal transition region lay between the lower critical Reynolds number of 2,030 and an upper critical value of 2,750. The dye experiments of Prengle and Rothfus indicated sinuous flow in the main stream at Reynolds numbers between 1,225 and 2,100. Large disturbance eddies were cast off with increased frequency as the Reynolds number was increased from 2,100 to 2,800. Above the latter value, both pressure-drop and dye studies suggested fully turbulent motion throughout the main stream.

In the case of concentric annuli as well as parallel plates the limits of the tran-

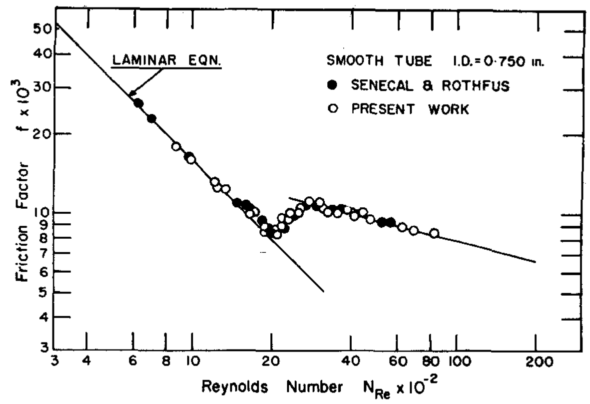

Fig. 1A.

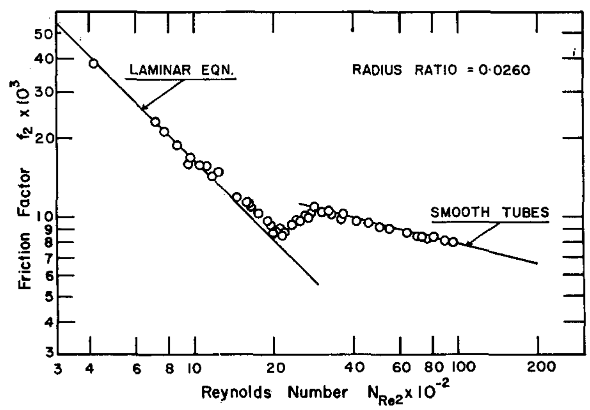

Fig. 1B.

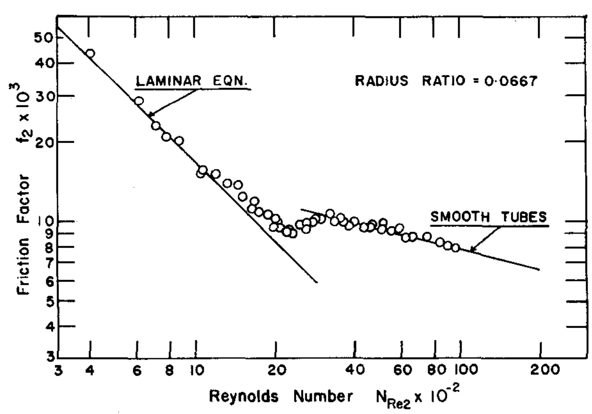

Fig. 1C.

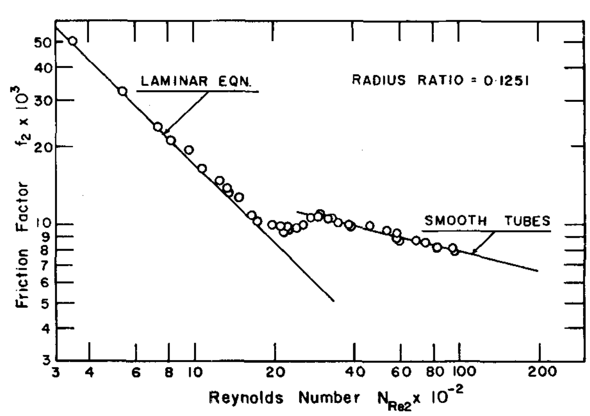

Fig. 1D.

Fig. 1. Fanning friction factors for tubes, parallel plates, and the outer surfaces of annuli.

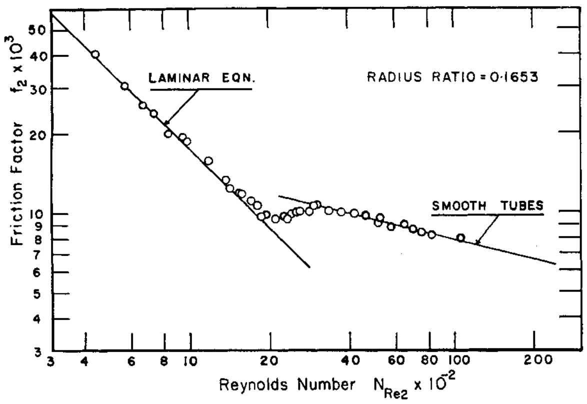

Fig. 1E.

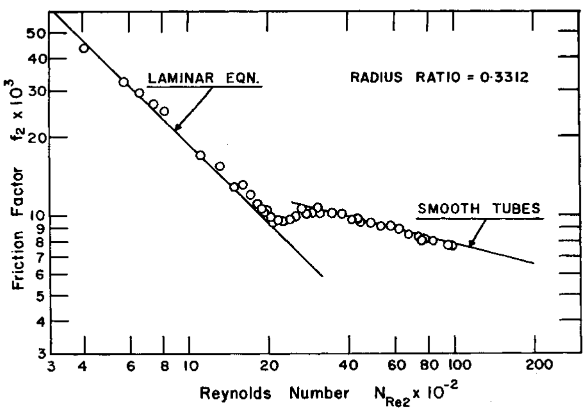

Fig. 1F.

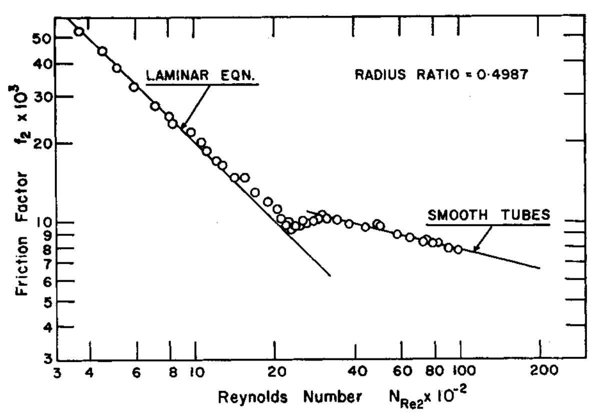

Fig. 1G.

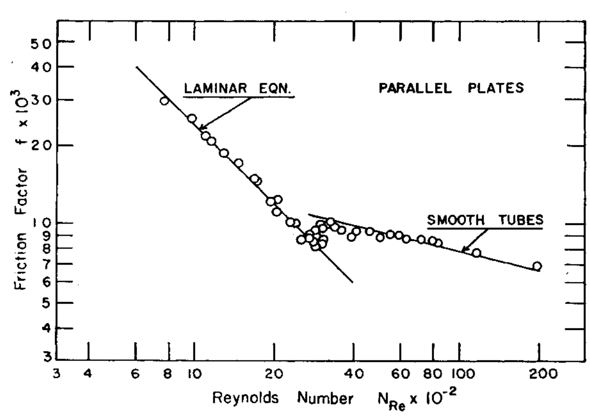

Fig. 1H.

Fig. 1 (*continued*).

TABLE 1

EFFECT OF GEOMETRY ON VISCOUS-FLOW FRICTION FACTORS IN CONCENTRIC ANNULI

r_1/r_2	0.0	0.1	0.2	0.4	0.6	0.8	1.0
$\psi_1(r_1/r_2)$, Equation (7)	1.00	1.40	1.44	1.48	1.49	1.50	1.50
$\psi_2(r_1/r_2)$, Equation (19)	1.00	1.06	1.11	1.21	1.31	1.41	1.50

sition region and the friction factor behavior within that region have not been established adequately. A few consistent data, such as Carpenter's, are available, but no single investigator has obtained precise data over a wide range of radius ratios. The dye studies of Prengle and Rothfus indicated a progressive spread of sinuous flow over the main stream in annuli at Reynolds numbers above 700. Although this behavior was similar to that in smooth tubes, the authors could not extend their data to establish the Reynolds number at which the first disturbance eddy was cast off.

Turbulent Flow

It is customary to deal with friction in fully turbulent flow through noncircular ducts by applying the hydraulic radius concept to the entire stream. In brief, this assumes that the friction factor in Equation (5) can be obtained from the smooth-tube correlation at the Reynolds number defined in Equation (4).

A force balance on the entire stream shows that if the skin friction is equal at all points on the wetted perimeter,

$$\frac{\Delta p g_0}{L} = \frac{\tau_0 g_0}{R_H} = \frac{4\tau_0 g_0}{D_e} \tag{8}$$

The balance can also be written in the form of Equation (5) through the introduction of the appropriate friction factor. If the skin friction varies from point to point around the perimeter of the conduit, the corresponding local friction factors must also change. The over-all friction factor of Equation (5) represents the integrated effect of the local values on the pressure drop and may bear a complex relationship to the physical situation. It is, therefore, rather unlikely that the hydraulic-radius method should be completely successful unless the stipulation of uniform skin friction is fulfilled. Parallel plates, however, meet this requirement and thus stand a good chance of being handled satisfactorily by means of the empirical procedure.

On the other hand, a force balance on the entire stream in an annular duct shows that

$$\frac{\Delta p g_0}{L} = \frac{2(r_1\tau_1 g_0 + r_2\tau_2 g_0)}{(r_2^2 - r_1^2)} \tag{9}$$

The two skin frictions therefore, would have to be averaged in order to reproduce the form of Equation (8). It seems unlikely that any more than a rough approximation of the actual pressure drop would be forthcoming from the over-all application of hydraulic radius.

At Reynolds numbers between 3,000 and 100,000 in smooth tubes the Fanning friction factor is closely correlated by the Blasius equation (*1*)

$$f = 0.079/(N_{Re})^{0.25} \tag{10}$$

Sage and coworkers (*3, 5, 6*) have obtained friction data for flow between parallel plates which are in approximate agreement with Equation (10) at Reynolds numbers from 6,960 to 53,200. It should be noted that infinite parallel plates are simulated in practice by rectangular passages of large-aspect ratio. It is exceptionally difficult to isolate conditions in the central portion of such ducts from side effects; therefore, it is not surprising that friction factors for parallel plates are generally measured with less precision than those for tubes.

Early measurements of friction in concentric annuli reviewed by Wiegand and Baker (*11*) show conflicting deviations from Equation (10) with varying radius ratios. More recent data on local velocities as well as friction (*4, 8*) have pointed toward a more consistent dependence of friction factor on the geometry of the conduit.

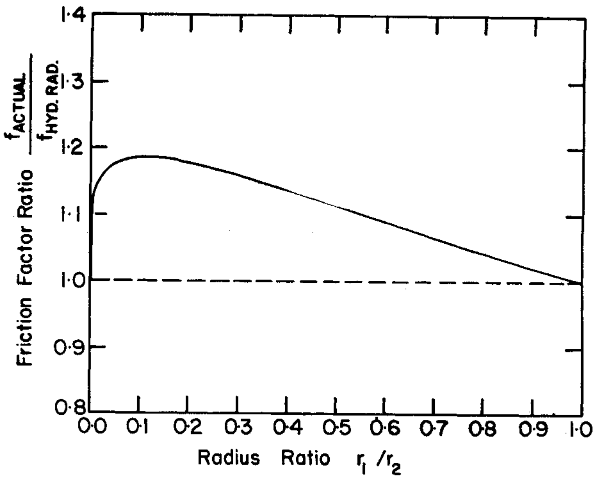

Fig. 2. Comparison of over-all friction factors in the lower turbulent range with those calculated by means of hydraulic radius applied to the whole annular stream.

EXPERIMENTAL EQUIPMENT

Static pressure gradients were measured in a brass tube and six concentric annuli formed by fitting the tube with metallic cores of various radii. Pressure drops were also obtained in a brass duct of rectangular cross section. Principal dimensions of the various conduits are summarized in Table 2. Upstream calming lengths varied from 167 to 450 equivalent diameters.

The test fluid was water at room temperature. Isothermal flow was maintained by means of heat exchangers installed in the external piping system through which the water was circulated steadily and continuously.

The pressure differences were indicated on vertical U-tube manometers designed to minimize contamination of the liquid-liquid interface. Use of a micromanometer was avoided through the selection of monochlorobenzene as a manometer fluid. Since the specific gravity of monochlorobenzene at 20°C. is 1.1084 referred to water at 20°C., the desired degree of multiplication was obtained in the manometer readings at low pressure differences. For larger differences in pressure, carbon tetrachloride was used as the manometer fluid.

BASIS OF CORRELATION

The steady, uniform, isothermal flow of an incompressible fluid through any smooth, concentric annulus will be considered, including the limiting cases of tubes and parallel plates. At some radial distance r_m from the center of the configuration, the velocity profile goes through a maximum point and the local shearing stress reaches zero. In fully viscous flow, the force balance on a differential element of fluid predicts that

$$r_m^2 = \frac{r_2^2 - r_1^2}{\ln\left(\frac{r_2}{r_1}\right)^2} \tag{11}$$

Katz and Knudsen (*4*) and Rothfus, Monrad, and Senecal (*8*) have shown by velocity measurements that the same relationship serves to represent r_m in fully turbulent flow as well as in the viscous range. Rothfus and coworkers (*9*) have used drag measurements at the core to show that the radius of maximum velocity does not, however, obey Equation (11) in the transition range. Exceptions to this are the limiting cases of tubes and parallel plates where symmetry forces the point of maximum velocity to remain undisturbed.

If a portion of the fluid lying between the radius of maximum velocity and an arbitrary radius r is considered, the hydraulic radius for this segment may be defined in the usual manner. The cross-sectional area occupied by the fluid is $\pi(r^2 - r_m^2)$ and the perimeter over which fluid shear is exerted is simply $2\pi r$, since no shear exists at the radius of maximum velocity. Thus

$$R_{H_r} = \frac{r^2 - r_m^2}{2r} \tag{12}$$

If r is taken to be r_1 or r_2, the radii of the inner and outer surfaces, respectively, the definite hydraulic radii R_{H_1} and R_{H_2} can be specified through Equation (12). For example,

$$R_{H_2} = \frac{r_2^2 - r_m^2}{2r_2} \tag{13}$$

It is apparent that R_{H_1} loses significance as the core is made vanishingly small. The portion of the fluid between the radius of maximum velocity and the outer surface of the annulus is of more immediate concern for purposes of correlation.

The pressure gradient due to friction is related to the skin friction, $\tau_2 g_0$, at the outer boundary through the force balance

$$\frac{\Delta p g_0}{L} = \frac{\tau_2 g_0}{R_{H_2}} = \frac{4\tau_2 g_0}{D_{e_2}} \tag{14}$$

which is the equivalent of Equation (8). By analogy with Equation (5), a friction factor f_2 for the outer surface can be defined in such manner as to make

$$\Delta p = \frac{2 f_2 \rho V^2 L}{4 g_0 R_{H_2}} = \frac{2 f_2 \rho V^2 L}{g_0 D_{e_2}} \tag{15}$$

TABLE 2

DIMENSIONS OF EXPERIMENTAL CONDUITS

Conduit	Outer tube Material	r_2, in.	Inner tube Material	r_1, in.	(r_1/r_2)	$r_2 - r_1$, in.	r_m, in.
Tube	brass	0.3750	—	—	0.0000	—	0.000
Annulus 1	brass	0.3750	Steel	0.0098	0.0260	0.365	0.139
2	brass	0.3750	Steel	0.0250	0.0667	0.350	0.161
3	brass	0.3750	Steel	0.0469	0.1251	0.328	0.182
4	brass	0.3750	Steel	0.0620	0.1653	0.313	0.195
5	brass	0.3750	Copper	0.1242	0.3312	0.251	0.238
6	brass	0.3750	Copper	0.1870	0.4987	0.188	0.276
Parallel plates*	brass	∞	Brass	∞	1.0000	0.700	

*Rectangular passage, 14 in. wide by 0.700 in. clearance.

It is, therefore, reasonable to investigate the merit of the hydraulic-radius method of correlating fully turbulent friction by applying the concept to that part of the fluid lying outside the radius of maximum velocity. Only in the cases of tubes and parallel plates is this the same as dealing with the entire stream.

Extending the hydraulic radius concept in this manner implies that the friction factor f_2 at the outer wall should be obtained from the smooth-tube correlation at the Reynolds number

$$N_{Re_2} = \frac{2(r_2^2 - r_m^2)V\rho}{r_2\mu} \qquad (16)$$

provided that the flow is fully turbulent. Thus at Reynolds numbers in the lower turbulent range,

$$f_2 = 0.079/(N_{Re_2})^{0.25} \qquad (17)$$

for annuli and parallel plates as well as tubes.

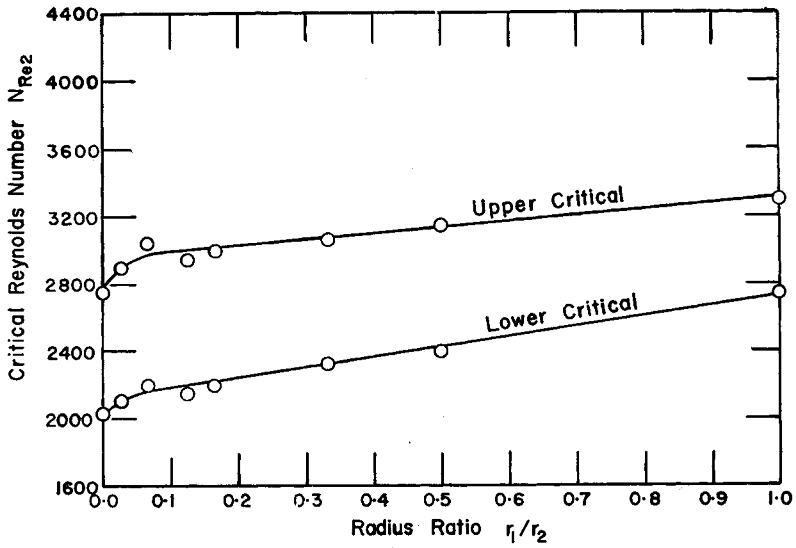

Fig. 3. Effect of radius ratio on critical Reynolds numbers.

RESULTS AND DISCUSSION

The experimental data* are summarized in Figure 1 as graphs of f_2 against N_{Re_2} on logarithmic coordinates. The abscissae have been calculated from Equation (16) and the ordinates from Equation (15). The radius of maximum velocity has been obtained from Equation (11) over the whole Reynolds-number range, since the behavior of r_m in the transition region is not well known.

Comparison of Equations (5) and (15) shows that

$$f_2 = f\left(\frac{D_{e_2}}{D_e}\right) = f\left(\frac{N_{Re_2}}{N_{Re}}\right) \qquad (18)$$

Therefore, in fully viscous flow, combination of Equations (7) and (18) yields the theoretical expression

$$f_2 = \frac{16}{N_{Re_2}}\,\psi_2\left(\frac{r_1}{r_2}\right) \qquad (19)$$

$$= \frac{16}{N_{Re_2}}\left[\frac{(r_2^2 - r_m^2)^2}{r_2^2(r_2^2 + r_1^2 - 2r_m^2)}\right]$$

Values of the bracketed term at various radius ratios are shown in Table 1. The experimental data are in exact agreement with Equation (19) at low Reynolds numbers where entirely viscous flow can reasonably be expected.

In the fully turbulent range the friction factors for all three cross sections obey Equation (17) very closely. The parallel-plate data deviate from the tube curve somewhat more than do the data for annuli. Since the experimental uncertainty is greater in the case of parallel plates, as previously mentioned, Equation (17) can still be taken to represent these data adequately. In addition to the wide range of radius ratios covered by the present data, Rothfus, Monrad, and Senecal have observed that their data for annuli having radius ratios of 0.1625 and 0.650 also obey Equation (17) in the lower turbulent range. It can, therefore, be concluded that a unique correlation for all concentric annuli is attainable through application of the hydraulic radius concept to the portion of the fluid lying between the outer boundary of the conduit and the radius of maximum velocity.

Since the friction factor at the outer surface is so accurately correlated in this manner, it is of interest to inquire what magnitude of error would be incurred by using the equivalent diameter for the whole stream in the usual way. In this case the over-all friction factor f, defined by Equations (5) and (6), is assumed to obey Equation (10) with the Reynolds number calculated by means of Equation (4). On the other hand, the actual values of f must be related to the experimental values of f_2 through Equation (18). Combination of Equations (10), (17), and (18) shows the actual over-all friction factor to be related to the friction factor predicted from over-all hydraulic radius by the equation

$$\frac{f\ \text{actual}}{f\ \text{hydraulic radius}} = \left[\frac{2\left(1 - \frac{r_1}{r_2}\right)\ln\frac{r_2}{r_1}}{\left(\frac{r_1}{r_2}\right)^2 - 1 + 2\ln\frac{r_2}{r_1}}\right]^{1.25} \qquad (20)$$

It is, therefore, apparent that the friction factor calculated by means of over-all hydraulic radius must differ from the actual friction factor by an amount dependent on the radius ratio of the conduit. There is also a small effect of Reynolds number since the value of the exponent in the last equation is related to the slope of the logarithmic friction-factor–Reynolds-number correlation.

The friction-factor ratio of Equation (20) is shown at various radius ratios in Figure 2. It is apparent that the error due to using the over-all hydraulic-radius method reaches a maximum value at $r_1/r_2 = 0.11$. Since many commercial annuli have radius ratios between 0.5 and 1.0, the error incurred may not be very serious if pressure drop alone is the important variable. On the other hand, at lower radius ratios or where precise friction data are required as a basis for other correlations, the application of hydraulic radius to the whole stream may not be sufficiently accurate. In any case the correction obtained from Equation (20) or Figure 2 very simply reduces the error to a negligible percentage in the lower turbulent range.

Even at Reynolds numbers above 100,000 it is recommended that f_2 be obtained from the smooth-tube correlation at N_{Re_2} and the pressure drop calculated by means of Equation (15). The friction factor f_1 at the inner surface, defined by Equation (15) with the equivalent diameter

$$De_1 = 4R_{H_1} = \frac{2(r_m^2 - r_1^2)}{r_1} \qquad (21)$$

replacing the equivalent diameter De_2, can easily be related to the friction factor f_2. As shown by Rothfus, Monrad, and Senecal,

*Complete tabular material has been deposited as document 5438 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $6.25 for photoprints or $2.50 for 35-mm. microfilm.

$$\frac{f_1}{f_2} = \frac{\tau_1 g_0}{\tau_2 g_0} = \frac{r_2\,(r_m^{\,2} - r_1^{\,2})}{r_1\,(r_2^{\,2} - r_m^{\,2})} \qquad (22)$$

The last equation is valid at all Reynolds numbers since it is nothing more than a combination of force balances. In the fully viscous and fully turbulent regions, the radius of maximum velocity can be obtained from Equation (11).

The friction factors shown in Figure 1 serve to mark the limits of the transition range. In every case a dip occurs in the correlations for the noncircular ducts in much the same manner as for smooth tubes. The upper and lower critical Reynolds numbers marking the maxima and minima in the transition friction factors are shown as functions of the radius ratio in Figure 3. Except for annuli having very small cores a linear relationship adequately represents the data.

It should be noted that the Reynolds numbers in Figure 3 are calculated with the radius of maximum velocity indicated in Equation (11). Rothfus and coworkers (9) have found that r_m is greater than the computed value at the lower critical Reynolds number. Therefore, the actual lower critical value is somewhat smaller than shown in Figure 3. The same appears to be true at the upper critical Reynolds number to a lesser degree. Any firm conclusion, however, must be postponed until accurate velocity distributions have been obtained experimentally.

The data of Figure 1 also indicate a small upward deviation from the viscous relationship at Reynolds numbers below the lower critical point. The behavior of the friction factors in this range of flow is similar to that in smooth tubes. The data are consistent with the results of previous dye studies, which indicate that a regime of sinuous flow proceeds the initial formation of disturbance eddies in annuli as well as in tubes.

The friction data suggest that the Reynolds number at which viscous flow first becomes unstable depends on the radius ratio. Although pressure drop is an insensitive indication of sinuous flow, there is some tendency for the first deviation from viscous behavior to appear at lower and lower Reynolds numbers as the radius ratio is increased. No conclusion can be drawn in the absence of velocity distributions, but it is likely that the Reynolds number of 700 reported by Prengle and Rothfus is only a rough approximation of the actual values at which sinuous flow begins in annular ducts.

In view of the experimental results and the foregoing discussion, it is possible to predict the frictional pressure drop for isothermal, steady, uniform flow through smooth concentric annuli with about the same precision as in the case of smooth tubes. If the flow is fully viscous or fully turbulent, the individual skin frictions at the inner and outer surfaces of the conduit can also be computed. If the fluid is in transitional flow, only the skin friction at the outer wall can be obtained from the present data. The limits of the transition region have been established experimentally over the whole range of radius ratios as shown in Figures 1 and 3.

In order to calculate pressure drops or individual skin frictions, the flow regime should be established by calculating the radius of maximum velocity from Equation (11) and the Reynolds number from Equation (16). Interpolation in Figure 1 then indicates whether viscous, transition, or turbulent flow can be expected to prevail under the predetermined conditions of operation. If the flow is entirely viscous, the friction factor at the outer wall f_2 can be obtained from Equation (19) and the pressure drop can then be calculated by means of Equation (15). The friction factor at the inner wall f_1 can be calculated, in turn, from Equation (22). Then the skin frictions at the two surfaces can be obtained separately from their defining equations

$$\tau_1 g_0 = \frac{f_1}{2}\,\rho V^2 \qquad (23)$$

and

$$\tau_2 g_0 = \frac{f_2}{2}\,\rho V^2 \qquad (24)$$

If the flow is fully turbulent, the outer wall friction factor f_2 can be obtained at the Reynolds number N_{Re_e} from Figure 1 or a similar graph of friction factor against Reynolds number for smooth tubes. The pressure drop can then be calculated from Equation (15), the inner wall friction factor from Equation (22), and the individual skin frictions from Equations (23) and (24). If the flow is transitional, the value of f_2 is best obtained by interpolating in Figure 1. The pressure drop can then be calculated by means of Equation (15).

NOTATION

- b = half clearance between parallel flat plates, ft.
- D = diameter of tube, ft.; D_1 = inner diameter of annular space, ft.; D_2 = outer diameter of annular space, ft.
- D_e = equivalent diameter of noncircular duct based on hydraulic radius of the whole stream, ft.; D_{e_1} and D_{e_2} = equivalent diameters based on hydraulic radii R_{H_1} and R_{H_2} respectively, ft.
- f = Fanning friction factor defined in Equations (1) and (5), dimensionless; f_1 and f_2 = friction factors for the inner and outer surfaces, respectively, of an annular conduit, dimensionless
- g_0 = conversion factor = 32.2 (lb. mass)(ft.)/(lb. force)(sec.^{2})
- L = length of conduit, ft.
- N_{Re} = Reynolds number defined in Equations (2) and (4), dimensionless; N_{Re_2} = Reynolds number for annuli defined in Equation (16), dimensionless
- Δp = pressure drop due to fluid friction, lb. force/sq. ft.
- r = radius from center of configuration to a point in the fluid stream, ft.; r_1 = inner radius of annular space, ft.; r_2 = outer radius of annular space, ft.
- r_m = radius from center of configuration to the point of maximum local fluid velocity, ft.
- R_H = hydraulic radius (i.e., cross-sectional area of fluid + wetted perimeter) for the whole stream, ft.; R_{H_1} = hydraulic radius of the fluid between r_m and r_1, ft.; R_{H_2} = hydraulic radius of the fluid between r_m and r_2, ft.; R_{H_r} = hydraulic radius of the fluid between r_m and r, ft.
- V = bulk average linear velocity of the fluid, ft./sec.

Greek Letters

- μ = viscosity of the fluid, lb. mass/(sec.)(ft.)
- ρ = density of the fluid, lb. mass/cu. ft.
- τ = local shearing stress at a point in the fluid stream, lb. force/sq. ft.; τ_0 = skin friction at the wall of a conduit, lb. force/sq. ft.; τ_1 and τ_2 = skin frictions at the inner and outer surfaces, respectively, of an annular space, lb. force/sq. ft.
- ψ_1, ψ_2 = geometrical functions defined in Equations (7) and (19), dimensionless

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